

# First Grade

2018-2019 Guide

September 10- November 16<sup>th</sup>

Eureka

Module 1: *Sum and Differences to 10*



ORANGE PUBLIC SCHOOLS  
OFFICE OF CURRICULUM AND INSTRUCTION  
OFFICE OF MATHEMATICS

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## Module 1 Performance Overview

- In Topic A, we continue the work of developing the ability to add fluently with all the numbers within 10 in “put together” (adding) situations, with a special focus on the numbers 6, 7, 8, and 9 in 5-group configurations (groups of 5). Students separate numbers into 2 visual sets and record their decompositions as number bonds.
- As students move into Topic B, they gain momentum with putting together, composing and decomposing, and counting on to determine the total. Students use both concrete and pictorial situations to describe all of the decompositions of 6, 7, 8, 9, and 10.
- In Topic C, students develop a more robust understanding of addition word problems, moving beyond kindergarten problems by reviewing put together with result unknown and add to with result unknown problems, and then moving to the more complex change unknown version of the earlier problem types.
- Topic D affords students the opportunity to solve problems within the simplicity of equations, moving on from the context of story problems. Students begin Topic D with tracking the number of counts on from a given number by using their fingers and 5-group cards.
- Topic E leads students to a very intentional understanding and application of the equal sign and the commutative property of addition.
- Topic G focuses on students understanding the meaning of subtraction as it relates to addition.
- With a smooth transition from Topic G, Topic H provides students with rich experiences connecting subtraction to their solid foundation of addition, by having conversations about how drawings relate to equations and represent their understanding of story problems.
- Topic I allows students to learn methods for subtraction which involve subtracting 0 and 1, subtracting the whole number, subtracting one less than the whole number, and using familiar decompositions (5-groups and partners of 10) to conceptualize subtraction as finding a missing part.
- Module 1 closes with Topic J, where students analyze the addition chart for repeated reasoning and structures that support their journey towards fluency within 10.



## Module 1: Sum and Differences to 10

### Pacing:

September 10<sup>th</sup> - November 16<sup>th</sup>

**42 Days**

Topic	Lesson	Lesson Objective/ Supportive Videos
<b>Topic A:</b> Embedded Numbers and Decompositions	Lesson 1	Analyze and Describe embedded numbers (to 10) using 5-groups and number bonds <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 2	Reason about embedded numbers in varied configurations using number bonds. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 3	See and describe numbers of objects using 1 more within 5-group configurations. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Topic B:</b> Counting On from Embedded Numbers	Lesson 4	Represent situations with number bonds. Count from one embedded number or part to totals of 6 and 7 and generate all addition expressions for each total. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 5	Represent put together situations with number bonds. Count on from one embedded number or part to totals of 6 and 7 and generate all addition expressions for each total. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 6	Represent situations with number bonds. Count on from one embedded number or part to totals of 8 and 9 and generate all expressions for each total. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 7	Represent situations with number bonds. Count on from one embedded number or part to totals of 8 and 9 and generate all expressions for each total. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 8	Represent all the number pairs of 10 as number bond diagrams from a given scenario and generate all expressions equal to 10. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Topic C:</b> Addition Word Problems	Lesson 9	Solve unknown math stories by drawing, writing equations, and making statements of the solution. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 10	Solve math stories by drawing and using 5-group cards. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 11	Solve add to with change unknown math stories as a context for counting by drawing, writing equations, and making statements of the solution. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>

	Lesson 12	Solve add to with change unknown math stories using 5-group cards. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 13	Tell put together with result unknown, add to with result unknown, and add to with change unknown stories from equations. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Topic D:</b> Strategies for Counting On	Lesson 14	Count on up to 3 more using numeral and 5-group cards and fingers to track the change. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 15	Count on up to 3 more using numeral and 5-group cards and fingers to track the change. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 16	Count on to find the unknown part in missing addend equations such as $6 + \_ = 9$ . Answer, “How many more to make 6, 7, 8, 9, and 10?” <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Topic E:</b> The Commutative Property of Addition and the Equal Sign	Lesson 17	Understand the meaning of the equal sign by pairing equivalent expressions and constructing true number sentences. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 18	Understand the meaning of the equal sign by pairing equivalent expressions and constructing true number sentences. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 19	Represent the same story scenario with addends repositioned (the commutative property). <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 20	Apply the commutative property to count on from a larger addend. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Topic F:</b> Development of Addition Fluency within 10	Lesson 21	Visualize and solve doubles and doubles plus 1 with 5-group cards. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 22	Look for and make use of repeated reasoning on the addition chart by solving and analyzing problems with common addends. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 23	Look for and make use of structure on the addition chart by looking for and coloring problems with the same total. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 24	Practice to build fluency with facts to 10. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Mid-Module Assessment Task</b> <b>(Interview Style)</b> <b>October 15-16</b>		

<p><b>Topic G:</b> Subtraction as an Unknown Addend &amp; <b>Topic H:</b> Subtraction Word Problems</p>	Lesson 25	Solve add to with change unknown math stories with addition and relate to subtraction. Model with materials and write corresponding number sentences. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 30	Solve add to with change unknown math stories with drawings, relating addition and subtraction. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 28/29	Solve take from with result unknown math stories with math drawings, true number sentences and statements, using horizontal marks to cross off what is taken away. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>  Solve take apart with addend unknown math stories with math drawings, equations, and statements, circling the known part to find the unknown. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 31	Solve take from with change unknown math stories with drawings. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 32	Solve put together/take apart with addend unknown math stories. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<p><b>Topic I:</b> Decomposition Strategies for Subtraction</p>	Lesson 33	Model 0 less and 1 less pictorially and as subtraction number sentences. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 34	Model $n - n$ and $n - (n - 1)$ pictorially and as subtraction sentences. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 35	Relate subtraction facts involving fives and doubles to corresponding decompositions. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 36	Relate subtraction from ten to corresponding decompositions. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 37	Relate subtraction from nine to corresponding decompositions. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<p><b>Topic J:</b> Development of Subtraction Fluency Within 10</p>	Lesson 38	Look for and make use of repeated reasoning and structure using the addition chart to solve subtraction problems. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 39	Analyze the addition chart to create sets of related addition and subtraction facts. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<p><b>End-of- Module Assessment Task (Interview Style: 3 days)</b> <b>November 15-16, 2018</b></p>		

## NJSLS Standards:

**1.OA.1**

Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Introduce situations for students to model- starting with concrete materials; and as students are ready- working with drawings, part-part- whole representations, and number lines.

Have a variety of materials for students to model the problems.

First grade students extend their experiences in Kindergarten by working with numbers to 20 to solve a new type of problem situation: Compare (See **Table 1** in this document for examples of all problem types). In a Compare situation, two amounts are compared to find “How many more” or “How many less”.

As students show proficiency with models and drawings, begin to demonstrate how to represent the actions using equations. Give students the opportunity to say the equation orally and then in writing.

The vocabulary of comparison situations can cause confusion for students. While the words *more than* implies addition and *fewer than* implies subtraction, that is not always the case. Avoid teaching “ Key Words”

Problem Type: Compare		
<p><u>Difference Unknown:</u> “How many more?” version. Lucy has 7 apples. Julie as 9 apples. How many more apples does Julie have than Lucy?</p>	<p><u>Bigger Unknown:</u> “More” version suggests operation. Julie has 2 more apples than Lucy. Lucy has 7 apples. How many apples does Julie have?</p>	<p><u>Smaller Unknown:</u> Version with “more”  Mastery expected in Second Grade</p>
<p>“How many fewer?” version Lucy as 7 apples. Julie has 9 apples. How many fewer apples does Lucy have than Julie?  <math>7 + \square = 9</math>  <math>9 - 7 = \square</math></p>	<p><u>Bigger Unknown:</u> Version with “fewer”  Mastery expected in Second Grade</p>	<p><u>Smaller Unknown:</u> “Fewer” version suggests operation. Lucy has 2 fewer apples than Julie. Julie has 9 apples. How many apples does Lucy have?</p>

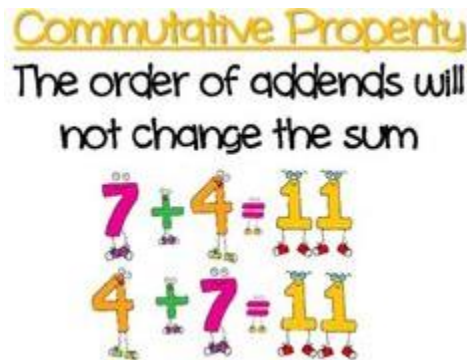


1.OA.3

Apply properties of operations as strategies to add and subtract. Examples: If  $8 + 3 = 11$  is known, then  $3 + 8 = 11$  is also known. (**Commutative property of addition.**) To add  $2 + 6 + 4$ , the second two numbers can be added to make a ten, so  $2 + 6 + 4 = 2 + 10 = 12$ . (**Associative property of addition.**) Students need not use formal terms for these properties.

Elementary students often believe that there are hundreds of isolated addition and subtraction facts to be mastered. However, when students understand the commutative and associative properties, they are able to use relationships between and among numbers to solve problems. First Grade students apply properties of operations as strategies to add and subtract. Students do not use the formal terms “commutative” and “associative”. Rather, they use the understandings of the commutative and associative property to solve problems.

Students use mathematical tools and representations (e.g., cubes, counters, number balance, number line, 100 chart) to model these ideas.



**Commutative Property Examples: Cubes**

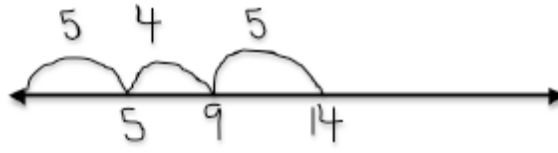
A student uses 2 colors of cubes to make as many different combinations of 8 as possible. When recording the combinations, the student records that 3 green cubes and 5 blue cubes equals 8 cubes in all. In addition, the student notices that 5 green cubes and 3 blue cubes also equals 8 cubes.



**Associative Property Examples: Number Line: □ = 5 + 4 + 5**

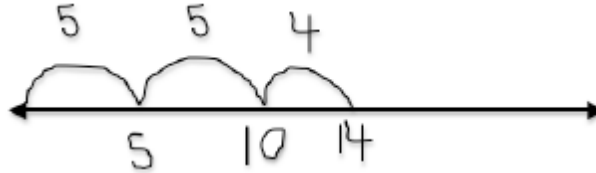
**Student A:** First I jumped to 5. Then, I jumped 4 more, so I landed on 9. Then I jumped 5 more and landed on 14.





**Student B:** I got 14, too, but I did it a different way. First I jumped to 5. Then, I jumped 5 again. That's 10.

Then, I jumped 4 more. See, 14!



**Mental Math:** **There are 9 red jelly beans, 7 green jelly beans, and 3 black jelly beans. How many jelly beans are there in all?**

**Student:** "I know that 7 + 3 is 10. And 10 and 9 is 19. There are 19 jelly beans."

Although subtraction is not commutative, it is important not to contribute to a potential misconception by saying that you cannot take a larger number from a smaller number. It is possible to take a larger number from a smaller number. This results in a negative number and this is learned in middle school.

1.OA.4

Understand subtraction as an unknown-addend problem. For example, subtract  $10 - 8$  by finding the number that makes 10 when added to 8.

First Graders often find subtraction facts more difficult to learn than addition facts. By understanding the relationship between addition and subtraction, First Graders are able to use various strategies described below to solve subtraction problems.

Utilize a part-part-whole representation to help students make the connection between addition and subtraction equations in different forms.

### **For Sums to 10**

#### Think-Addition:

Think-Addition uses known addition facts to solve for the unknown part or quantity within a problem. When students use this strategy, they think, "What goes with this part to make the total?" The think-addition strategy is particularly helpful for subtraction facts with sums of 10 or less and can be used for sixty-four of the 100 subtraction facts. Therefore, in order for think-addition to be an effective strategy, students must have mastered addition facts first.

For example, when working with the problem  $9 - 5 = \square$ , First Graders think "Five and what makes nine?", rather than relying on a counting approach in which the student counts 9, counts off 5, and then counts what's left. When subtraction is presented in a way that encourages students to think using addition, they use known addition facts to solve a problem.

### **For Sums Greater than 10**

The 36 facts that have sums greater than 10 are often considered the most difficult for students to master. Many students will solve these particular facts with *Think-Addition* (described above), while other students may use other strategies described below. Regardless of the strategy used, all strategies focus on the relationship between addition and subtraction and often use 10 as a benchmark number.

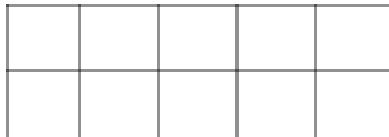
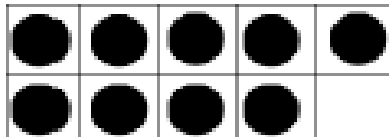
#### **Build Up Through 10:**

This strategy is particularly helpful when one of the numbers to be subtracted is 8 or 9. Using 10 as a bridge, either 1 or 2 are added to make 10, and then the remaining amount is added for the final sum.

Example:  $15 - 9 = \square$

**Student A:** "I'll start with 9. I need one more to make 10. Then, I need 5 more to make 15. That's 1 and 5- so it's 6.  $15 - 9 = 6$ ."

**Student B:** "I put 9 counters on the 10 frame. Just looking at it I can tell that I need 1 more to get to 10. Then I need 5 more to get to 15. So, I need 6 counters."



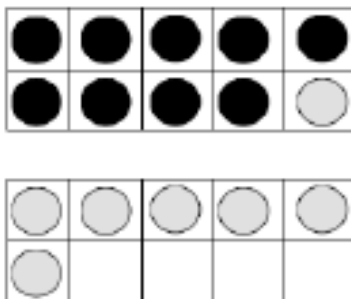
#### **Back Down Through 10**

This strategy uses take-away and 10 as a bridge. Students take away an amount to make 10, and then take away the rest. It is helpful for facts where the ones digit of the two-digit number is close to the number being subtracted.

Example:  $16 - 7 = \square$

**Student A:** "I'll start with 16 and take off 6. That makes 10. I'll take one more off and that makes 9.  $16 - 7 = 9$ ."

**Student B:** “I used 16 counters to fill one ten frame completely and most of the other one. Then, I can take these 6 off from the 2nd ten frame. Then, I’ll take one more from the first ten frame. That leaves 9 on the ten frame.”



**1.OA.5**

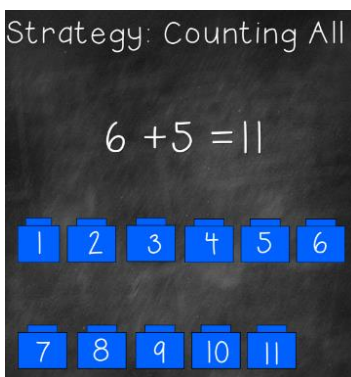
Relate counting to addition and subtraction within 20.

When solving addition and subtraction problems to 20, First Graders often use counting strategies, such as counting all, counting on, and counting back, before fully developing the essential strategy of using 10 as a benchmark number. Once students have developed counting strategies to solve addition and subtraction problems, it is very important to move students toward strategies that focus on composing and decomposing number using ten as a benchmark number, as discussed in 1.OA.6, particularly since counting becomes a hindrance when working with larger numbers. By the end of First Grade, students are expected to use the strategy of 10 to solve problems.

While introducing a strategy may be a whole lesson, individualized activities that allow students to progress through the strategies at a rate determined by their understanding is important.

**Counting All (addition)**

Start with 1 and count to find the total number of objects



**Counting On & Counting Back (addition):** Students hold a “start number” in their head and count on/back from that number.

Example:  $15 + 2 = \square$

Example:  $12 - 3 = \square$

Counting On

Holding 15 in her head, the student holds up one finger and says 16, then holds up another finger and says 17. The student knows that  $15 + 2$  is 17, since she counted on 2 using her fingers.

Counting Back

Keeping 12 in his head, the student counts backwards, “11” as he holds up one finger; says “10” as he holds up a second finger; says “9” as he holds up a third finger. Seeing that he has counted back 3 since he is holding up 3 fingers, the student states that  $12 - 3 = 9$ .

**1.OA.6**

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g.,  $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$ ); decomposing a number leading to a ten (e.g.,  $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$ ); using the relationship between addition and subtraction (e.g., knowing that  $8 + 4 = 12$ , one knows  $12 - 8 = 4$ ); and creating equivalent but easier or known sums (e.g., adding  $6 + 7$  by creating the known equivalent  $6 + 6 + 1 = 12 + 1 = 13$ ).

In First Grade, students learn about and use various strategies to solve addition and subtraction problems. When students repeatedly use strategies that make sense to them, they internalize facts and develop fluency for addition and subtraction within 10. When students are able to demonstrate fluency within 10, they are accurate, efficient, and flexible. First Graders then apply similar strategies for solving problems within 20, building the foundation for fluency to 20 in Second Grade.

Continue to watch for students who are double counting when adding an/or subtracting.

**Developing Fluency for Addition & Subtraction within 10**

Example: **Two frogs were sitting on a log. 6 more frogs hopped there. How many frogs are sitting on the log now?**

Counting- On

I started with 6 frogs and then counted up, Sixxxx... 7, 8. So there are 8 frogs on the log.

$$6 + 2 = 8$$

Internalized Fact

There are 8 frogs on the log. I know this because 6 plus 2 equals 8.

$$6 + 2 = 8$$

**Add and Subtract within 20**

Example: **Sam has 8 red marbles and 7 green marbles. How many marbles does Sam have in all?**

Making 10 and Decomposing a Number

I know that 8 plus 2 is 10, so I broke up (decomposed) the 7 up into a 2 and a 5. First I added 8 and 2 to get 10, and then added the 5 to get 15.

$$\begin{aligned} 7 &= 2 + 5 \\ 8 + 2 &= 10 \\ 10 + 5 &= 15 \end{aligned}$$

Creating an Easier Problem with Known Sums

I broke up (decomposed) 8 into 7 and 1. I know that 7 and 7 is 14. I added 1 more to get 15.

$$\begin{aligned} 8 &= 7 + 1 \\ 7 + 7 &= 14 \\ 14 + 1 &= 15 \end{aligned}$$

Example: **There were 14 birds in the tree. 6 flew away. How many birds are in the tree now?**

Back Down Through Ten

I know that 14 minus 4 is 10. So, I broke the 6 up into a 4 and a 2. 14 minus 4 is 10. Then I took away 2 more to get 8.

$$\begin{aligned} 6 &= 4 + 2 \\ 14 - 4 &= 10 \\ 10 - 2 &= 8 \end{aligned}$$

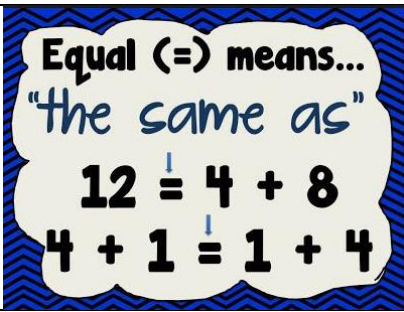
Relationship between Addition & Subtraction

I thought, '6 and what makes 14?'. I know that 6 plus 6 is 12 and two more is 14. That's 8 altogether. So, that means that 14 minus 6 is 8.

$$\begin{aligned} 6 + 8 &= 14 \\ 14 - 6 &= 8 \end{aligned}$$

**1.OA.7**

Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false?  $6 = 6$ ,  $7 = 8 - 1$ ,  $5 + 2 = 2 + 5$ ,  $4 + 1 = 5 + 2$ .



In order to determine whether an equation is true or false, First Grade students must first understand the meaning of the equal sign. This is developed as students in Kindergarten and First Grade solve numerous joining and separating situations with mathematical tools, rather than symbols. Once the concepts of joining, separating, and “the same amount/quantity as” are developed concretely, First Graders are ready to connect these experiences to the corresponding symbols (+, -, =). Thus, students learn that the equal sign does not mean “the answer comes next”, but that the symbol signifies an equivalent relationship that the left side ‘has the same value as’ the right side of the equation.

When students understand that an equation needs to “balance”, with equal quantities on both sides of the equal sign, they understand various representations of equations, such as:

- an operation on the left side of the equal sign and the answer on the right side ( $5 + 8 = 13$ )
- an operation on the right side of the equal sign and the answer on the left side ( $13 = 5 + 8$ )
- numbers on both sides of the equal sign ( $6 = 6$ )
- operations on both sides of the equal sign ( $5 + 2 = 4 + 3$ ).

Once students understand the meaning of the equal sign, they are able to determine if an equation is true ( $9 = 9$ ) or false ( $9 = 8$ ).

It is appropriate in early experiences using the equal sign to have students read it as, “*is the same as.*”

**1.OA.8**

Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations.  
 $8 + ? = 11$        $5 = \square - 3$        $6 + 6 = \square$

First Graders use their understanding of and strategies related to addition and subtraction as described in 1.OA.4 and 1.OA.6 to solve equations with an unknown. Rather than symbols, the unknown symbols are boxes or pictures.

**Example: Five cookies were on the table. I ate some cookies. Then there were 3 cookies. How many cookies did I eat?**

**Student A:** What goes with 3 to make 5? 3 and 2 is 5. So, 2 cookies were eaten.

**Student B:** Five, four, three (*holding up 1 finger for each count*). 2 cookies were eaten (*showing 2 fingers*).

**Student C:** We ended with 3 cookies. Three, four, five (*holding up 1 finger for each count*). 2 cookies were eaten (*showing 2 fingers*).

Example: **Determine the unknown number that makes the equation true.  $5 - \square = 2$**

**Student:** 5 minus something is the same amount as 2. Hmmm. 2 and what makes 5? 3!

So, 5 minus 3 equals 2.

Now it's true!

I can find the missing digit in an addition or subtraction sentence with 3 numbers.

$$3 + 3 = \underline{\quad\quad} \quad 7 + \underline{\quad\quad} = 14 \quad 12 = \underline{\quad\quad} + 6$$

$$3 + \underline{\quad\quad} = 8 \quad 3 + \underline{\quad\quad} = 9 \quad 10 = 4 + \underline{\quad\quad}$$

$$\underline{\quad\quad} + 9 = 12 \quad 5 + \underline{\quad\quad} = 10 \quad 12 = \underline{\quad\quad} + 5$$

$$10 - 2 = \underline{\quad\quad} \quad 8 - \underline{\quad\quad} = 3 \quad \underline{\quad\quad} - 6 = 4$$

$$16 - 8 = \underline{\quad\quad} \quad 15 - 8 = \underline{\quad\quad} \quad \underline{\quad\quad} - 4 = 8$$

$$3 = 9 - \underline{\quad\quad} \quad 8 = 9 - \underline{\quad\quad} \quad \underline{\quad\quad} = 6 - 4$$

M : Major Content

S: Supporting Content

A : Additional Content



Common addition and subtraction.<sup>1</sup>

	<b>RESULT UNKNOWN</b>	<b>CHANGE UNKNOWN</b>	<b>START UNKNOWN</b>
<b>ADD TO</b>	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
<b>TAKE FROM</b>	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	<b>TOTAL UNKNOWN</b>	<b>ADDEND UNKNOWN</b>	<b>BOTH ADDENDS UNKNOWN<sup>2</sup></b>
<b>PUT TOGETHER / TAKE APART<sup>3</sup></b>	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$ , $5 - 3 = ?$	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? $5 = 0 + 5$ , $5 + 0 = 5$ , $1 + 4$ , $5 = 4 + 1$ , $5 = 2 + 3$ , $5 = 3 + 2$
<b>COMPARE</b>	<b>DIFFERENCE UNKNOWN</b>	<b>BIGGER UNKNOWN</b>	<b>SMALLER UNKNOWN</b>
	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$ , $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ , $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ , $? + 3 = 5$

<sup>1</sup> Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

<sup>2</sup> These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the  $-$  sign does not always mean, makes or results in but always does mean is the same number as.

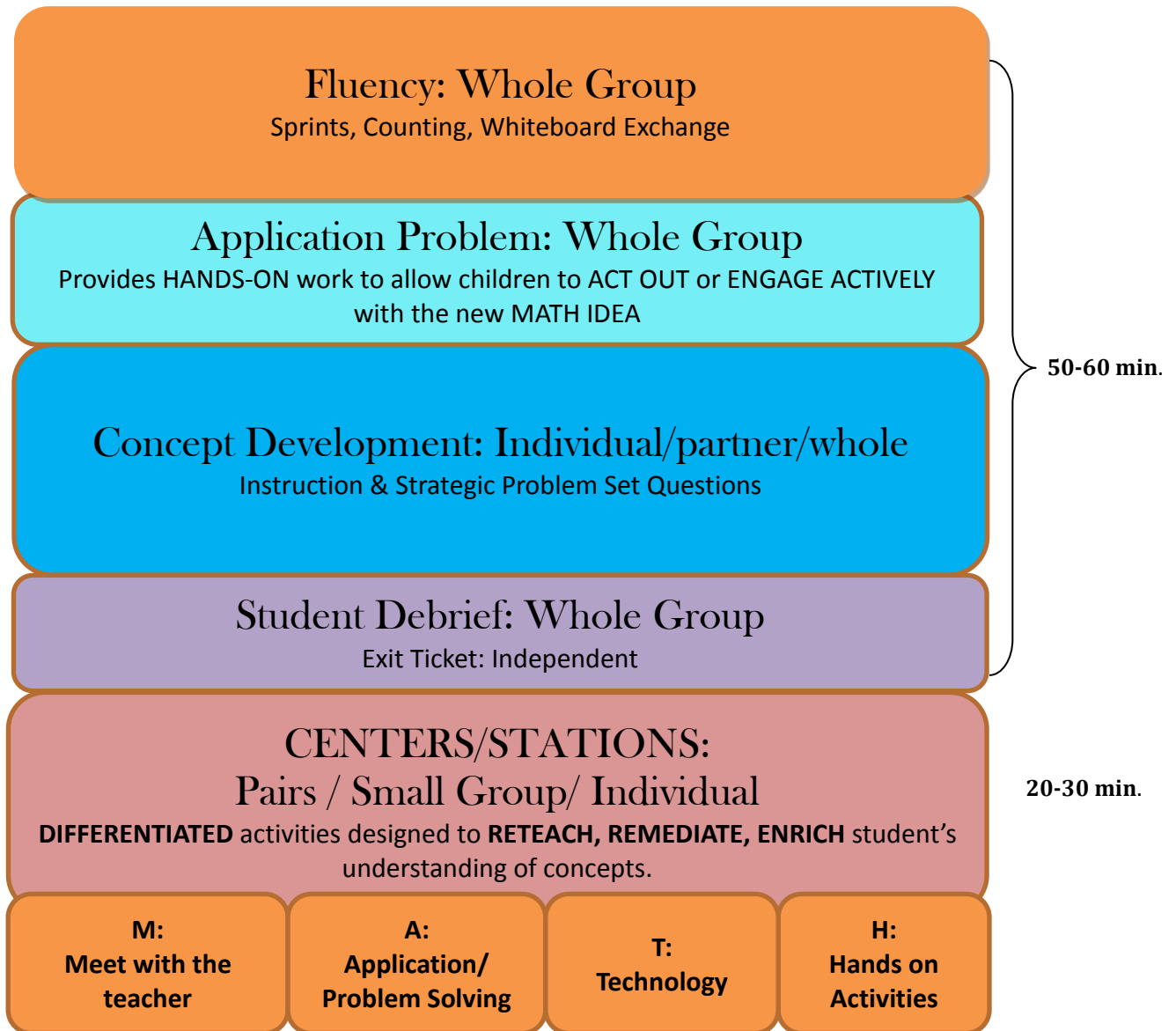
<sup>3</sup> Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

<sup>4</sup> For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

<http://www.corestandards.org/Math/Content/mathematics-glossary/Table-1/>

<b>Module 1 Assessment / Authentic Assessment Recommended Framework</b>			
<b>Assessment</b>	<b>CCSS</b>	<b>Estimated Time</b>	<b>Format</b>
Diagnostic Assessment (IREADY)		1-2 blocks	Individual
<b><u>Eureka Math</u></b> <b><u>Module 1: Sums and Differences to 10</u></b>			
Portfolio/Authentic Assessment #1	1.OA.7	30 mins	Individual
Optional Mid Module Assessment (Interview Style)	1.OA.1,3-8	1 Block	Individual or Small Group with Teacher
<i>Grade 1 Interim 1/ Unit Assessment (Interview Style)</i>	1.OA.1,3-8	½ block	Individual or Small Group with Teacher
Optional End of Module Assessment (Interview Style)	1.OA.1,3-8	1 Block	Individual or Small Group with Teacher

# First Grade Ideal Math Block



## Eureka Lesson Structure:

### Fluency:

- Sprints
- Counting: Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models
- Whiteboard Exchange

### Application Problem:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

### Concept Development: (largest chunk of time)

#### Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

#### Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

### Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

## PARCC Assessment Evidence/Clarification Statements

CCSS	Evidence Statement	Clarification	Math Practices
1.OA.A.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknown in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	i) Tasks should include all problem situations and all of their subtypes and language variants. Mastery is expected in “Add To” and “Take From” - Result and Change Unknown Problems, “Put Together/Take Apart” Problems, “Compare” – Difference Unknown, Bigger Unknown (more version) and Smaller Unknown (fewer version) Problems (for more information see CCSS Table 1 and OA Progression, p. 9.) ii) Interviews (individual or small group) are used to assess mastery of different problem types.	MP 1, 4
1.OA.B.3	Apply properties of operations as strategies to add and subtract. Examples: if $8+3 = 11$ is known, then $3+8 = 11$ is also known (Commutative property of addition). To add $2+6+4$ , the second two numbers can be added to make a ten, so $2+6+4 = 2+10 = 12$ (Associative property of addition).	i) Tasks should not expect students to know the names of the properties. ii) Interviews (individual or small group) should target students’ application of properties of operations to add and subtract.	MP 7,8
1.OA.D.7	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? $6 = 6$ , $7 = 8-1$ , $5+2=2+5$ , $4+1=5+2$ .	i) Interviews (individual or small group) should target students’ understanding of the equal sign.	MP 7,8
1.OA.D.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8+?=11$ , $5=?-3$ , $6+6=?$ .	i) Interviews (individual or small group) should target students’ thinking strategies for determining the unknown in an addition or subtraction equation relating 3 whole numbers. Thinking strategies expected in Grade 1 (Level 2 and 3) are defined in 1.OA.6 and in OA Progression	MP 7,8

## Number Talks

### **What does Number Talks look like?**

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

### **Communication**

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

### **Mental Math**

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

### **Thumbs Up**

- This is just a signal to let you know that you have given your students enough time to think about the problem
- It will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

### **Teacher as Recorder**

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

### **Purposeful Problems**

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

### **Starting Number Talks in your Classroom**

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

### **The teacher asks questions:**

- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?
- What was the first thing your eyes saw, or your brain did?

Student Name: \_\_\_\_\_

Task: \_\_\_\_\_

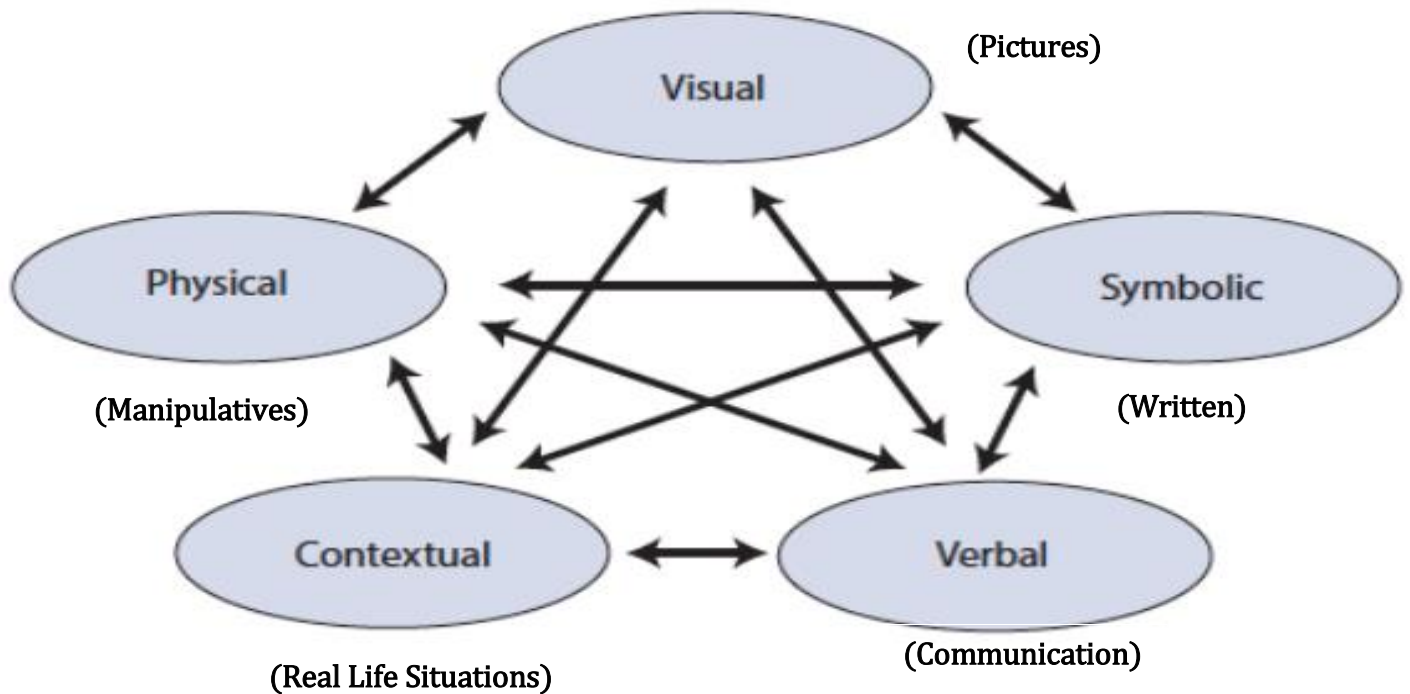
School: \_\_\_\_\_

Teacher: \_\_\_\_\_ Date: \_\_\_\_\_

"I CAN....."	STUDENT FRIENDLY RUBRIC				SCORE
	...a start 1	...getting there 2	...that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a classmate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my thinking.	



## Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

**Visual:** When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

**Physical:** The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

**Verbal:** Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

**Symbolic:** Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

**Contextual:** A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

### **The Lesh Translation Model: Importance of Connections**

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

## Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

**Concrete:** “Doing Stage”: Physical manipulation of objects to solve math problems.

**Pictorial:** “Seeing Stage”: Use of imaged to represent objects when solving math problems.

**Abstract:** “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

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## Read, Draw, Write Process

**READ** the problem. Read it over and over.... And then read it again.

**DRAW** a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

**WRITE** your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

## Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

### Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?

The most  
important thing  
is to NEVER  
stop  
questioning

*Albert Einstein*

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

**100** questions that promote  
**Mathematical Discourse**

Help students **work together** to make sense of mathematics

- 1 What **strategy** did you use?
- 2 Do you **agree**?
- 3 Do you **disagree**?
- 4 Would you **ask the rest of the class** that question?
- 5 Could you **share your method** with the class?
- 6 What part of what he said **do you understand**?
- 7 Would someone like to **share** \_\_\_?
- 8 Can you **convince the rest of us** that your answer makes sense?
- 9 **What do others think** about what [student] said?
- 10 Can someone **retell or restate** [student]'s explanation?
- 11 Did you **work together**? In what way?
- 12 Would anyone like to **add to what was said**?
- 13 Have you **discussed** this with your group? With others?
- 14 Did anyone get a **different answer**?
- 15 **Where** would you go for **help**?
- 16 **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
- 17 How could you help another student **without telling them the answer**?
- 18 **How would you explain** \_\_\_ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is **mathematically correct**

- 19 Is this a **reasonable answer**?
- 20 Does that make **sense**?
- 21 **Why** do you think that? Why is that true?
- 22 Can you **draw a picture or make a model** to show that?
- 23 **How** did you reach that conclusion?
- 24 Does anyone want to **revise** his or her answer?
- 25 **How were you sure** your answer was right?

Ready

## Help students learn to reason mathematically

- 26 How did you **begin** to think about this problem?
- 27 What is **another way** you could solve this problem?
- 28 How could you **prove** \_\_\_\_\_?
- 29 Can you **explain how your answer is different from or the same as** [student]'s answer?
- 30 Let's **break the problem into parts**. What would the parts be?
- 31 Can you **explain this part more specifically**?
- 32 Does that **always work**?
- 33 Can you think of a case where that **wouldn't work**?
- 34 How did you **organize** your information? Your thinking?

## Help students with problem comprehension

- 39 What is this problem about? What can you **tell me about it**?
- 40 Do you need to **define or set limits** for the problem?
- 41 How would you **interpret** that?
- 42 Could you **reword that in simpler terms**?
- 43 Is there something that can be **eliminated** or that is **missing**?
- 44 Could you **explain** what the problem is asking?
- 45 What **assumptions** do you have to make?
- 46 What do you **know** about this part?
- 47 Which words were **most important**? Why?

## Help students evaluate their own processes and engage in productive peer interaction

- 35 What do you need to do **next**?
- 36 What have you **accomplished**?
- 37 What are your **strengths and weaknesses**?
- 38 Was your **group participation appropriate and helpful**?



## Help students learn to **conjecture, invent, and solve problems**

- 48 What would happen if \_\_\_?
- 49 Do you see a **pattern**?
- 50 What are some **possibilities** here?
- 51 Where could you find the **information** you need?
- 52 How would you **check your steps** or your answer?
- 53 What **did not work**?
- 54 How is your solution method the **same as or different from** [student]'s method?
- 55 Other than retracing your steps, **how can you determine** if your answers are appropriate?
- 56 How did you **organize** the information? Do you have a **record**?
- 57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?
- 58 What have you tried? What **steps** did you take?
- 59 How would it look if you used this **model** or these **materials**?
- 60 How would you draw a **diagram or make a sketch** to solve the problem?
- 61 Is there **another possible answer**? If so, explain.
- 62 Is there **another way to solve** the problem?
- 63 Is there **another model** you could use to solve the problem?
- 64 Is there anything you've **overlooked**?
- 65 **How did you think** about the problem?
- 66 What was your **estimate or prediction**?
- 67 How **confident** are you in your answer?
- 68 **What else** would you like to know?
- 69 What do you think comes **next**?
- 70 Is the solution **reasonable**, considering the context?
- 71 Did you have a **system**? Explain it.
- 72 Did you have a **strategy**? Explain it.
- 73 Did you have a **design**? Explain it.





## Help students learn to connect mathematics, its ideas, and its application

- 74 What is the **relationship** between \_\_\_ and \_\_\_?
- 75 Have we ever solved a problem **like this before**?
- 76 What uses of mathematics did you find in the **newspaper** last night?
- 77 What is the **same**?
- 78 What is **different**?
- 79 Did you use skills or build on concepts that were **not necessarily mathematical**?
- 80 Which **skills or concepts** did you use?
- 81 What **ideas** have we explored before that were useful in solving this problem?

- 82 Is there a **pattern**?
- 83 **Where else** would this strategy be useful?
- 84 How does this **relate** to \_\_\_?
- 85 Is there a **general rule**?
- 86 Is there a **real-life situation** where this could be used?
- 87 How would your method work with **other problems**?
- 88 What other problem does this seem to **lead to**?

### Help students persevere

- 95 What was **one thing you learned** (or two, or more)?
- 96 Did you **notice any patterns**? If so, describe them.
- 97 What **mathematics topics** were used in this investigation?
- 98 What were the **mathematical ideas** in this problem?
- 99 What is mathematically **different about these two situations**?
- 100 What are the **variables** in this problem? What stays **constant**?

- 89 Have you tried making a **guess**?
- 90 **What else** have you tried?
- 91 Would **another method** work as well or better?
- 92 Is there **another way** to draw, explain, or say that?
- 93 Give me another **related problem**. Is there an easier problem?
- 94 How would you **explain** what you know right now?

### Help students focus on the mathematics from activities

## **Conceptual Understanding**

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

## **Procedural Fluency**

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

## **Math Fact Fluency: Automaticity**

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

### **K-2 Math Fact Fluency Expectation**

**K.OA.5** Add and Subtract within 5.

**1.OA.6** Add and Subtract within 10.

**2.OA.2** Add and Subtract within 20.

## **Math Fact Fluency: Fluent Use of Mathematical Strategies**

First and second grade students are expected to solve addition and subtraction facts using a variety of strategies fluently.

**1.OA.6** Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Use strategies such as:

- counting on; making ten (e.g.,  $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$ );
- decomposing a number leading to a ten (e.g.,  $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$ );
- using the relationship between addition and subtraction; and
- creating equivalent but easier or known sums.

**2.NBT.7** Add and subtract within 1000, using concrete models or drawings and strategies based on:

- place value,
- properties of operations, and/or
- the relationship between addition and subtraction;

## Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

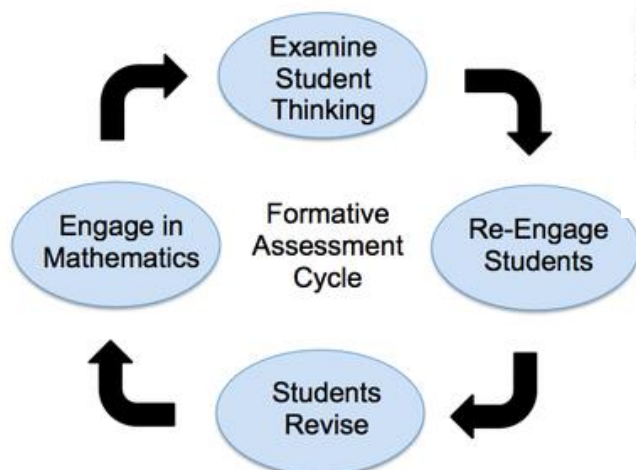
## Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

## Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.



*Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.*

(William 2007, pp. 1054; 1091)

## Connections to the Mathematical Practices

### **Student Friendly Connections to the Mathematical Practices**

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The **Standards for Mathematical Practice** describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

#### **Make sense of problems and persevere in solving them**

1 Mathematically proficient students in First Grade continue to develop the ability to focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere (Copley, 2010). As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, First Grade students become conscious of what they know and how they solve problems. They make sense of task-type problems, find an entry point or a way to begin the task, and are willing to try other approaches when solving the task. They ask themselves, “Does this make sense?” First Grade students’ conceptual understanding builds from their experiences in Kindergarten as they continue to rely on concrete manipulatives and pictorial representations to solve a problem, eventually becoming fluent and flexible with mental math as a result of these experiences..

#### **Reason abstractly and quantitatively**

2 Mathematically proficient students in First Grade recognize that a number represents a specific quantity. They use numbers and symbols to represent a problem, explain thinking, and justify a response. For example, when solving the problem: “There are 60 children on the playground. Some children line up. There are 20 children still on the playground. How many children lined up?” first grade students may write  $20 + 40 = 60$  to indicate a Think-Addition strategy. Other students may illustrate a counting-on by tens strategy by writing  $20 + 10 + 10 + 10 + 10 = 60$ . The numbers and equations written illustrate the students’ thinking and the strategies used, rather than how to simply compute, and how the story is decontextualized as it is represented abstractly with symbols.

### Construct viable arguments and critique the reasoning of others

3

Mathematically proficient students in First Grade continue to develop their ability to clearly express, explain, organize and consolidate their math thinking using both verbal and written representations. Their understanding of grade appropriate vocabulary helps them to construct viable arguments about mathematics. For example, when justifying why a particular shape isn't a square, a first grade student may hold up a picture of a rectangle, pointing to the various parts, and reason, "It can't be a square because, even though it has 4 sides and 4 angles, the sides aren't all the same size." In a classroom where risk-taking and varying perspectives are encouraged, mathematically proficient students are willing and eager to share their ideas with others, consider other ideas proposed by classmates, and question ideas that don't seem to make sense.

### Model with mathematics

4

Mathematically proficient students in First Grade model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. They also use tools, such as tables, to help collect information, analyze results, make conclusions, and review their conclusions to see if the results make sense and revising as needed.

### Use appropriate tools strategically

5

Mathematically proficient students in First Grade have access to a variety of concrete (e.g. 3-dimensional solids, ten frames, number balances, number lines) and technological tools (e.g., virtual manipulatives, calculators, interactive websites) and use them to investigate mathematical concepts. They select tools that help them solve and/or illustrate solutions to a problem. They recognize that multiple tools can be used for the same problem- depending on the strategy used. For example, a child who is in the counting stage may choose connecting cubes to solve a problem. While, a student who understands parts of number, may solve the same problem using ten-frames to decompose numbers rather than using individual connecting cubes. As the teacher provides numerous opportunities for students to use educational materials, first grade students' conceptual understanding and high-order thinking skills are developed

6	<p><b>Attend to precision</b></p> <p>Mathematically proficient students in First Grade attend to precision in their communication, calculations, and measurements. They are able to describe their actions and strategies clearly, using grade-level appropriate vocabulary accurately. Their explanations and reasoning regarding their process of finding a solution becomes more precise. In varying types of mathematical tasks, first grade students pay attention to details as they work. For example, as students' ability to attend to position and direction develops, they begin to notice reversals of numerals and self-correct when appropriate. When measuring an object, students check to make sure that there are not any gaps or overlaps as they carefully place each unit end to end to measure the object (iterating length units). Mathematically proficient first grade students understand the symbols they use (<math>=</math>, <math>&gt;</math>, <math>&lt;</math>), a proficient student who is able to attend to precision states, "Four is more than 3" rather than "The alligator eats the four. It's bigger."</p>
7	<p><b>Look for and make use of structure</b></p> <p>Mathematically proficient students in First Grade carefully look for patterns and structures in the number system and other areas of mathematics. For example, while solving addition problems using a number balance, students recognize that regardless whether you put the 7 on a peg first and then the 4, or the 4 on first and then the 7, they both equal 11 (commutative property). When decomposing two-digit numbers, students realize that the number of tens they have constructed 'happens' to coincide with the digit in the tens place. When exploring geometric properties, first graders recognize that certain attributes are critical (number of sides, angles), while other properties are not (size, color, orientation)</p>
8	<p><b>Look for and express regularity in repeated reasoning</b></p> <p>Mathematically proficient students in First Grade begin to look for regularity in problem structures when solving mathematical tasks. For example, when adding three one-digit numbers and by making tens or using doubles, students engage in future tasks looking for opportunities to employ those same strategies. Thus, when solving <math>8+7+2</math>, a student may say, "I know that 8 and 2 equal 10 and then I add 7 more. That makes 17. It helps to see if I can make a 10 out of 2 numbers when I start." Further, students use repeated reasoning while solving a task with multiple correct answers. For example, in the task "There are 12 crayons in the box. Some are red and some are blue. How many of each could there be?" First Grade students realize that the 12 crayons could include 6 of each color (<math>6+6 = 12</math>), 7 of one color and 5 of another (<math>7+5 = 12</math>), etc. In essence, students repeatedly find numbers that add up to 12.</p>



## Effective Mathematics Teaching Practices

**Establish mathematics goals to focus learning.** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

**Implement tasks that promote reasoning and problem solving.** Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

**Use and connect mathematical representations.** Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

**Facilitate meaningful mathematical discourse.** Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

**Pose purposeful questions.** Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

**Build procedural fluency from conceptual understanding.** Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

**Support productive struggle in learning mathematics.** Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

**Elicit and use evidence of student thinking.** Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.



## **5 Practices for Orchestrating Productive Mathematics Discussions**

Practice	Description/ Questions
1. Anticipating	<p>What strategies are students likely to use to approach or solve a challenging high-level mathematical task?</p> <p>How do you respond to the work that students are likely to produce?</p> <p>Which strategies from student work will be most useful in addressing the mathematical goals?</p>
2. Monitoring	<p>Paying attention to what and how students are thinking during the lesson.</p> <p>Students working in pairs or groups</p> <p>Listening to and making note of what students are discussing and the strategies they are using</p> <p>Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)</p>
3. Selecting	<p>This is the process of deciding the <i>what</i> and the <i>who</i> to focus on during the discussion.</p>
4. Sequencing	<p>What order will the solutions be shared with the class?</p>
5. Connecting	<p>Asking the questions that will make the mathematics explicit and understandable.</p> <p>Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.</p>

## MATH CENTERS/ WORKSTATIONS

*Math workstations* allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

### Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated.** If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.



In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.

In her book, she leads you step-by-step through the process of implementing work stations.

**MATH WORKSTATION INFORMATION CARD**

**Math Workstation:** \_\_\_\_\_

**Time:**

**NJSLS:**

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

**Objective(s):** By the end of this task, I will be able to:

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

**Task(s):**

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

**Exit Ticket:**

- \_\_\_\_\_
- \_\_\_\_\_
- \_\_\_\_\_

# MATH WORKSTATION SCHEDULE

Week of: \_\_\_\_\_

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	<b>BASED ON CURRENT OBSERVATIONAL DATA</b>
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	
	Group ____	Group ____	Group ____	Group ____	

## INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

## First Grade PLD Rubric

<b>Got It</b>		<b>Not There Yet</b>		
Evidence shows that the student essentially has the target concept or big math idea.		Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
<b>PLD Level 5: 100% Distinguished command</b>	<b>PLD Level 4: 89% Strong Command</b>	<b>PLD Level 3: 79% Moderate Command</b>	<b>PLD Level 2: 69% Partial Command</b>	<b>PLD Level 1: 59% Little Command</b>
<p>Student work shows <b>distinct levels of understanding</b> of the mathematics.</p> <p>Student <b>constructs</b> and <b>communicates</b> a <b>complete response</b> based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes an <b>efficient and logical progression</b> of mathematical reasoning and understanding.</p>	<p>Student work shows <b>strong levels of understanding</b> of the mathematics.</p> <p>Student <b>constructs</b> and <b>communicates</b> a <b>complete response</b> based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes a <b>logical progression</b> of mathematical reasoning and understanding.</p>	<p>Student work shows <b>moderate levels of understanding</b> of the mathematics.</p> <p>Student <b>constructs</b> and <b>communicates</b> a <b>complete response</b> based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes a <b>logical but incomplete progression</b> of mathematical reasoning and understanding. Contains <b>minor errors</b>.</p>	<p>Student work shows <b>partial understanding</b> of the mathematics.</p> <p>Student <b>constructs</b> and <b>communicates</b> an <b>incomplete response</b> based on student's attempts of explanations/ reasoning using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes an <b>incomplete or illogical progression</b> of mathematical reasoning and understanding.</p>	<p>Student work shows <b>little understanding</b> of the mathematics.</p> <p>Student <b>attempts to construct</b> and <b>communicates</b> a response using the:</p> <ul style="list-style-type: none"> <li>• Tools: <ul style="list-style-type: none"> <li>○ Manipulatives</li> <li>○ Five Frame</li> <li>○ Ten Frame</li> <li>○ Number Line</li> <li>○ Part-Part-Whole Model</li> </ul> </li> <li>• Strategies: <ul style="list-style-type: none"> <li>○ Drawings</li> <li>○ Counting All</li> <li>○ Count On/Back</li> <li>○ Skip Counting</li> <li>○ Making Ten</li> <li>○ Decomposing Number</li> </ul> </li> <li>• Precise use of math vocabulary</li> </ul> <p>Response includes <b>limited evidence of the progression</b> of mathematical reasoning and understanding.</p>
<b>5 points</b>	<b>4 points</b>	<b>3 points</b>	<b>2 points</b>	<b>1 point</b>

## DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

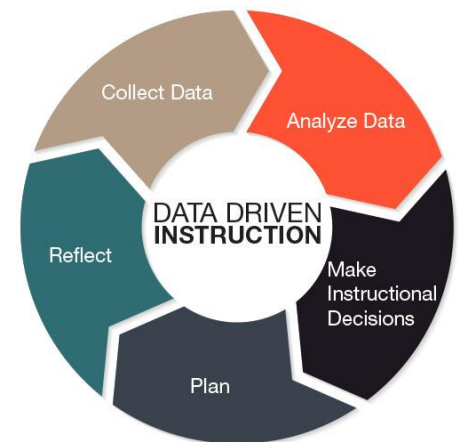
Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?

Now it is time to begin the analysis again.



**Data Analysis Form**

School: \_\_\_\_\_

Teacher: \_\_\_\_\_

Date: \_\_\_\_\_

Assessment: \_\_\_\_\_

NJSLS: \_\_\_\_\_

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

## **MATH PORTFOLIO EXPECTATIONS**

The **Student Assessment Portfolios for Mathematics** are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSL and be “practice forward” (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

### **K-2 GENERAL PORTFOLIO EXPECTATIONS:**

- Tasks contained within the Student Assessment Portfolios are “practice forward” and denoted as “Individual”, “Partner/Group”, and “Individual w/Opportunity for Student Interviews<sup>1</sup>.”
- Each Student Assessment Portfolio should contain a “Task Log” that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity “as a new and separate score” in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended*.
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)<sup>2</sup>.
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

## **GRADES K-2**

### **Student Portfolio Review**

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students should retain ALL of their current artifacts in their Mathematics Portfolio



**True or False**

Look at each equation. Tell if the equation is true or false by circling the word. Explain your reasoning with pictures, numbers, or words.

$$4 + 1 = 5 + 2$$

True

False

$$3 + 8 = 8 + 3$$

True

False

$$9 = 11 - 2$$

True

False

$$8 = 7 - 1$$

True

False

1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false?  $6 = 6$ ,  $7 = 8 - 1$ ,  $5 + 2 = 2 + 5$ ,  $4 + 1 = 5 + 2$ .

Mathematical Practices:

Individual

<b>Level 5: Distinguished Command</b>	<b>Level 4: Strong Command</b>	<b>Level 3: Moderate Command</b>	<b>Level 2: Partial Command</b>	<b>Level 1: No Command</b>
<p>Student correctly answers and clearly constructs and communicates a complete response based on explanations/reasoning using :</p> <ul style="list-style-type: none"> <li>• Concepts of equality</li> <li>• Strategies based the relationship between addition and subtraction</li> </ul> <p>Response includes an <b>efficient</b> and logical progression of steps.</p> <p>Strategy and execution meet the content, process, and qualitative demands of the task or concept. Student can communicate ideas.</p>	<p>Student correctly answers and clearly constructs and communicates a complete response with one minor calculation error based on explanations/reasoning using:</p> <ul style="list-style-type: none"> <li>• Concepts of equality</li> <li>• Strategies based the relationship between addition and subtraction</li> </ul> <p>Response includes a <b>logical</b> progression of steps</p> <p>May have minor errors that do not impact the mathematics.</p>	<p>Student answers, clearly constructs, and communicates a complete response with minor calculation errors based on explanations/reasoning using:</p> <ul style="list-style-type: none"> <li>• Concepts of equality</li> <li>• Strategies based the relationship between addition and subtraction</li> </ul> <p>Response includes a <b>logical but incomplete</b> progression of steps. Minor calculation errors.</p>	<p>The task is attempted and some mathematical effort is made. There may be fragments of accomplishment but little success.</p> <ul style="list-style-type: none"> <li>• Concepts of equality</li> <li>• Strategies based the relationship between addition and subtraction</li> </ul> <p>Response includes an <b>incomplete or illogical</b> progression of steps.</p>	<p>The student shows no work or justification. Further teaching is required.</p>

## Resources

### **Great Minds**

<https://greatminds.org/>

### **Embarc**

<https://embarc.online/>

### **Engage NY**

[http://www.engageny.org/video-library?f\[0\]=im\\_field\\_subject%3A19](http://www.engageny.org/video-library?f[0]=im_field_subject%3A19)

### **Common Core Tools**

<http://commoncoretools.me/>

<http://www.ccsstoolbox.com/>

<http://www.achievethecore.org/steal-these-tools>

### **Achieve the Core**

<http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12>

### **Manipulatives**

<http://nlvm.usu.edu/en/nav/vlibrary.html>

<http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000>

<http://www.thinkingblocks.com/>

**Illustrative Math Project** : <http://illustrativemathematics.org/standards/k8>

**Inside Mathematics:** <http://www.insidemathematics.org/index.php/tools-for-teachers>

**Sample Balance Math Tasks:** <http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/>

**Georgia Department of Education:** <https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx>

**Gates Foundations Tasks:** <http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf>

### **Minnesota STEM Teachers' Center:**

<http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships>

**Singapore Math Tests K-12:** <http://www.misskoh.com>

**Mobymax.com:** <http://www.mobymax.com>

# 21st Century Career Ready Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9. Model integrity, ethical leadership and effective management.
- CRP10. Plan education and career paths aligned to personal goals.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

For additional details see **21<sup>st</sup> Century Career Ready Practices** .

## References

“Eureka Math” *Great Minds*. 2018 < <https://greatminds.org/account/products>>